A new optimal double periodical construction of one target two-dimensional arrays

Oscar Moreno[1] Solomon Golomb

I. Introduction

One of the results of [3] is an answer to a question of [5] , and we in fact prove how the concept of one dimensional modular rulers, or they also called OOC's with correlation λ is the same as two dimensional double-periodic arrays with correlation λ. Using this we obtain the signal patterns for multiple targets. . Now using the previous mentioned result and a new construction of OOC's with $\lambda = 1$, we obtain a construction for multiple target arrays with auto and cross correlation one. This is the first family for multiple target arrays with $\lambda = 1$, and the arrays are of the extended sonar type . Furthermore given our equivalence theorem mentioned above (between two dimensional arrays and OOC's), and the fact that the new construction is optimal in the Johnson bound, we therefore obtain an optimal family of extended sonars with the property of double periodicity.
Our main result of this paper is theorem 2 below.

II. OOC's is the same concept as that of double-periodic multiple targets arrays

Let $A = [A(i,j)]$ and $B = [B(i,j)]$ be $r \times s$ matrices having $0, 1$ entries where r and s are relatively prime. We now have the following definition.

Definition 1 *The double-periodic cross-correlation between A and B is $\leq \lambda$ if*

$$\sum_{i=0}^{r-1}\sum_{j=0}^{s-1} A(i \oplus_r \alpha, j \oplus_s \tau)B(i,j) \leq \lambda$$

for any $\alpha \leq r, \tau \leq s$, where $\oplus_m$ denotes addition mod$m$ *the smallest such λ is the correlation. Auto-correlatiom is the same, but $A = B$. Let $a(.)$ and $b(.)$ be the sequences of length rs associated with the matrices A and B respectively via the chinese remainder theorem, i.e. $a(L) = A(L \pmod r, L \pmod s)$ and similarly $f(L) = B(L \pmod r, L \pmod s)$ for all $L, 0 \leq L \leq rs - 1$*

We now have the following theorem:

Theorem 1 *The collection of one-dimensional periodic auto and cross-correlation values of a family of sequences of length rs is precisely the same as the set of two dimensional doubly-periodic auto-and cross-correlation values of $r \times s$ matrices associated with these sequences via the residue map.*

Proof: This follows from the linearity of the residue map, for any τ, $0 \leq \tau \leq rs - 1$ and $\tau' = \tau \pmod r$, $\tau'' = \tau \pmod s$ as follows

$$\sum_{c=0}^{rs-1} a(C \oplus_{rs} \tau)f(C) = \sum_{i=0}^{r-1}\sum_{j=0}^{s-1} A \oplus_r \tau', j \oplus_s \tau'')B(i,j)$$

[1]This work was supported by NSF CISE Grant No. EIA-0080926

COR 1: The concept of an OOC with auto and cross correlation λ is the same as that of double periodic multi-target arrays with auto and cross-correlation λ.

COR 2: OOC's optimal in the Johnson Bound provide optimal multiple target double-periodic arrays.

COR 3: The OOC's of [4] provides many families of multiple target arrays.

REMARK: We also apply our result to the constructions of OOC's given in [2] and obtain new construction of signal patterns for multiple targets with auto-correlation 2 and cross-correlation 1. This is the first family with such correlation properties. Notice that the family might have more than 1 dot per row as well as per column and they are more general than the class of extended sonars. Note that the results just mentioned are quite surprising to us . One of the reasons is that it was proven that there does not exist Costas arrays with $n > 4$ with cross-correlation 1. Note that the quadratic sonars have auto-correlation 1 and cross-correlation 2, but we researched for many years trying to find arrays with auto-correlation 2 and cross-correlation 1, without success. For many practical reasons this kind of arrays should be useful. Maybe some researchers will be motivated to implement these arrays.

III. Main Result

In [3] we presented the results of our previous section.Using now our main theorem of the last section and the Singer OOC with parameters (l^4+l^2+1, l^2+1), where l is a prime power, we obtain a new two dimensional doubly periodical construction with parameters $((l^2 + l + 1)(l^2 - l + 1), l^2 + 1)$. This gives us the theorem:

Theorem 2 *For l a power of a prime using the Singer Construction we obtain an optimal doubly periodic array $(l^2 + l + 1) \times (l^2 - l + 1)$ with $l^2 + 1$ dots and auto correlation 1.*

Remark . The goal to the more general type of construction, as defined in [5] is to put as many dots as possible in our array. Given the optimally of the Singer OOC in the Johnson Bound our arrays will have the maximun number od dots in a doubly periodical array.

References

[1] S. W. Golomb and H. Taylor, Two-Dimensional Synchronization Patterns for Minimum Ambiguity, *IEEE IT Trans.* **IT-28, No. 4** (July 1982) 600–604.

[2] Guu-Chang Yang and Thomas E. Fuja. "Optical Orthogonal Codes with unequal auto and cross-correlation constraints". IEEE IT Transactions, Vol 41, No1, Jan 1995, pp 96-106

[3] Oscar Moreno and Solomon Golomb,"Optical Orthogonal Codes and Multi-Target Costas and Sonar Arrays" Presented at the IEEE-ISIT 2002, Lausanne, Switzerland.

[4] O. Moreno, Z. Zhiang, P. Kumar. and V. Zinoviev, "New constructions of optimal cyclically permutable constant weight codes." in IEEE Transactions on Information Theory, vol. 41. no. 2, 1995.

[5] S. W. Golomb and H. Taylor, "Two -Dimensional Synchronization Patterns for Minimum Ambiguity", *IEEE Transactions on Information Theory*, 28:4 (1982), pp. 600–604.